Approximate Boolean Operations on Free-form Solid

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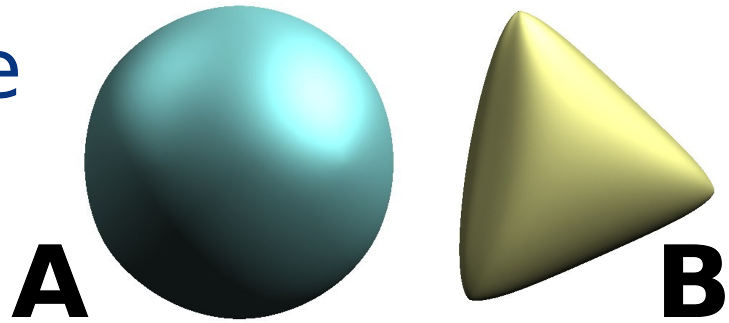
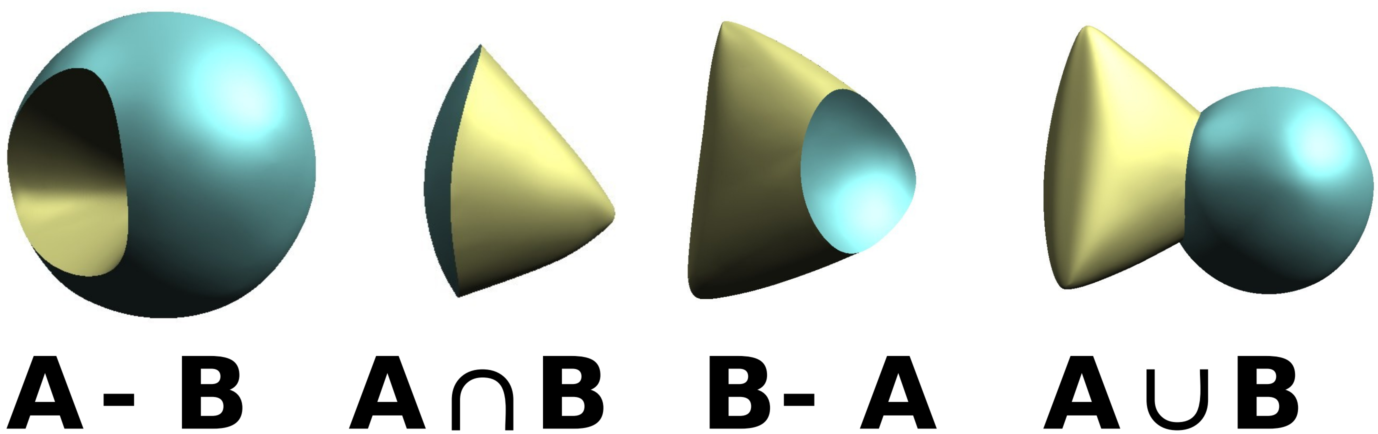
Boolean operations

Construct objects from parts

- combine



- difference, intersection, union



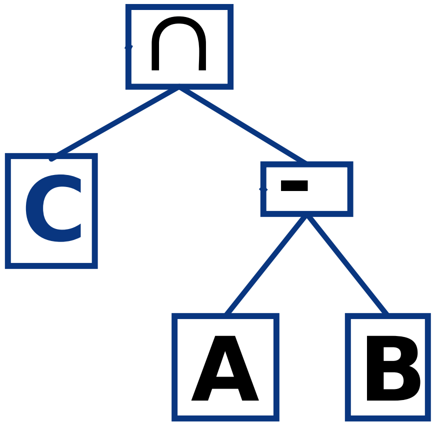
Surfaces and CSG

Constructive solid geometry

- solids: boolean expressions

  combine solids naturally

  not efficient for rendering, collisions, ...



Parameteric surfaces

- efficient representation for

  rendering, multiresolution, ...

- patches, subdivision, hierarchical

Approach

Approximate Boolean ops

- input: free-form solids as multires subdivision surfaces
- output: multires subdivision surface approximating the result

Want

- good approximation
- coarse control meshes
- user controlled precision

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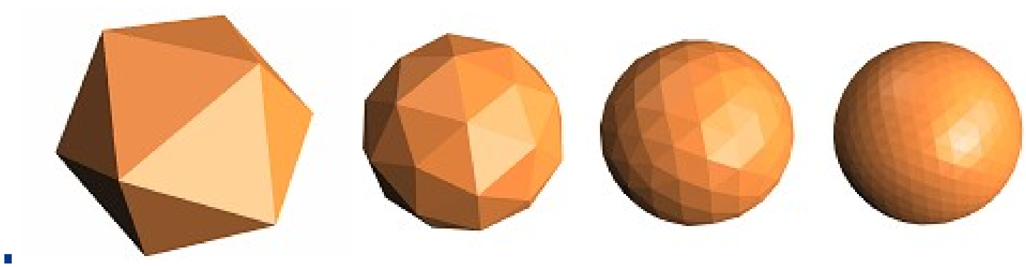
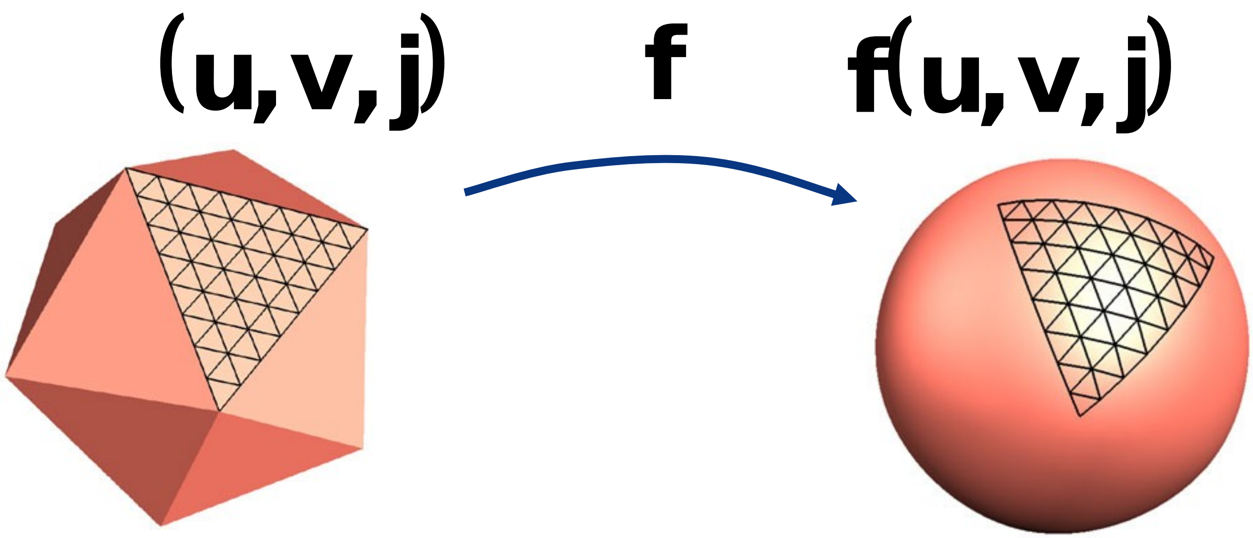
Background

Multires subdivision surfaces

- recursive refinement
- parameterization





domain geometry

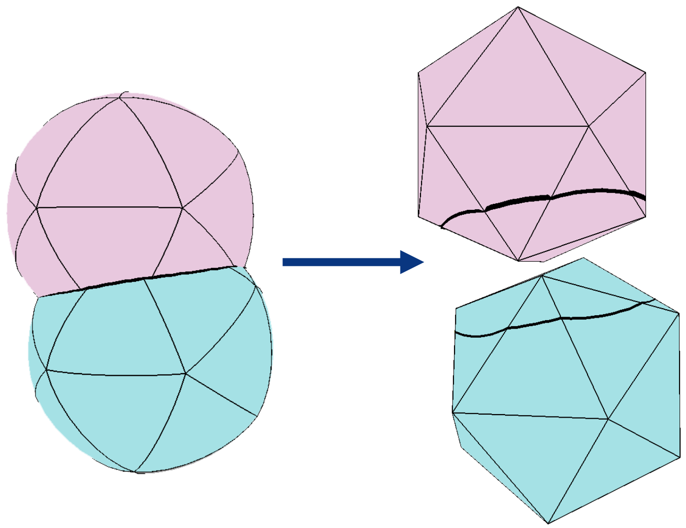
Related work

- solid modeling: too many to enumerate
- surface-surface intersection: too many...
- reparameterization: Eck et al. 95, Krishnamurthy 96, Lee et al. 98,...
- mesh optimization: Freitag et al.
- merging control meshes: Linsen97

Overview

4 main steps

- approximate intersection
- cut and merge meshes
- parameterization
- fitting



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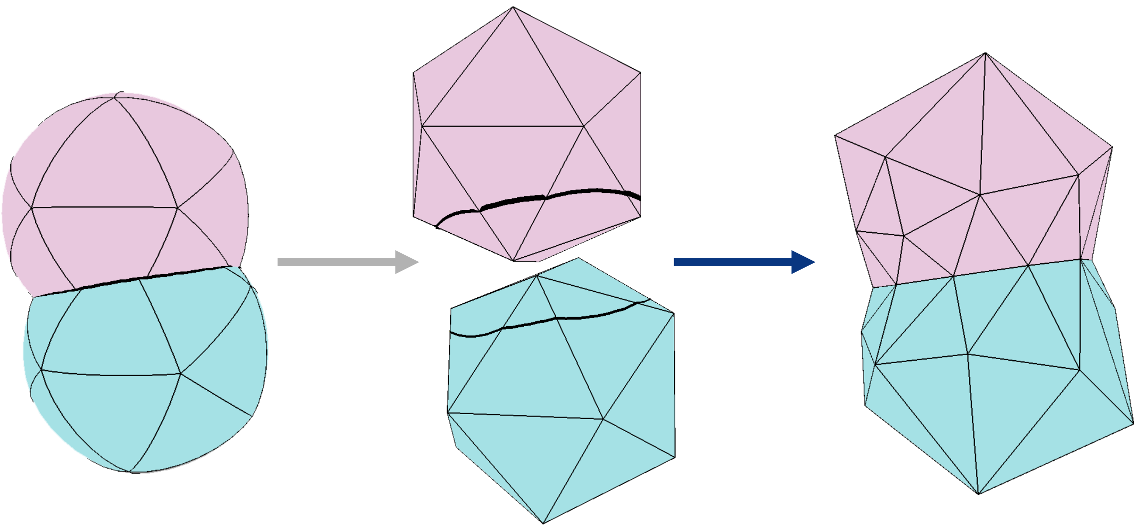

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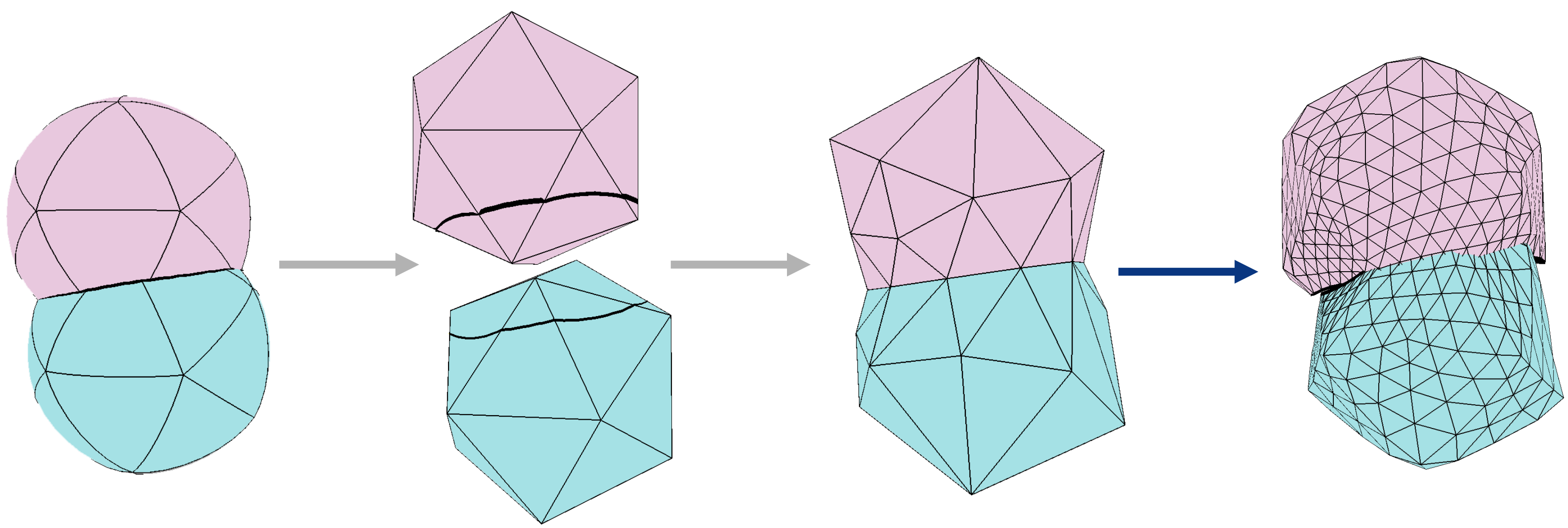

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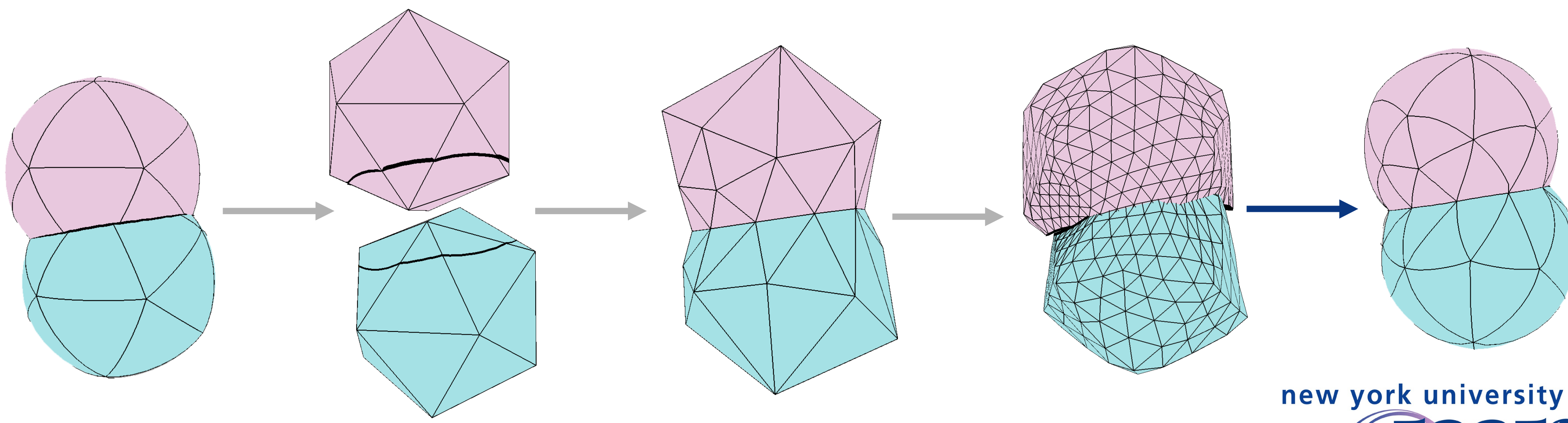

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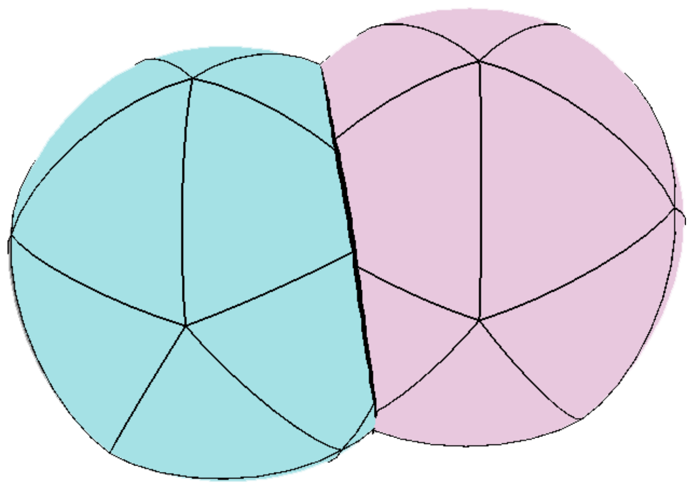
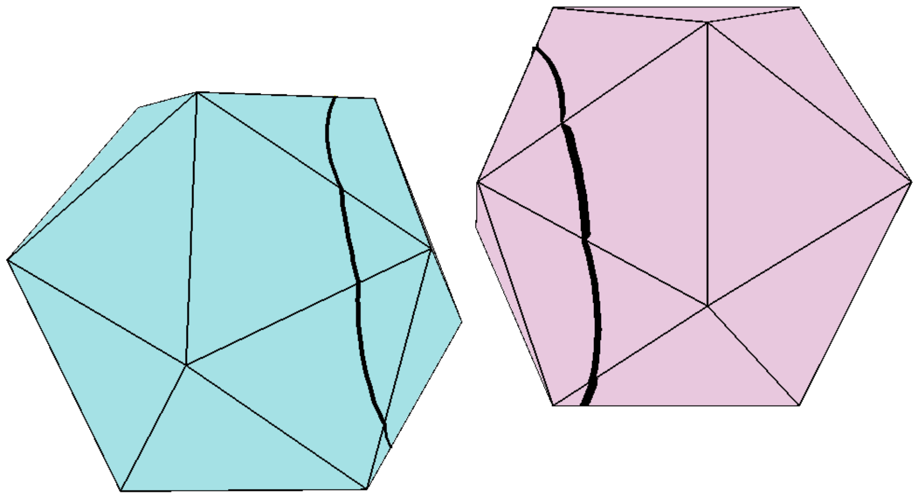
Surface intersection

Find intersection

- world-space location
- parametric location



world-space



parameter domain

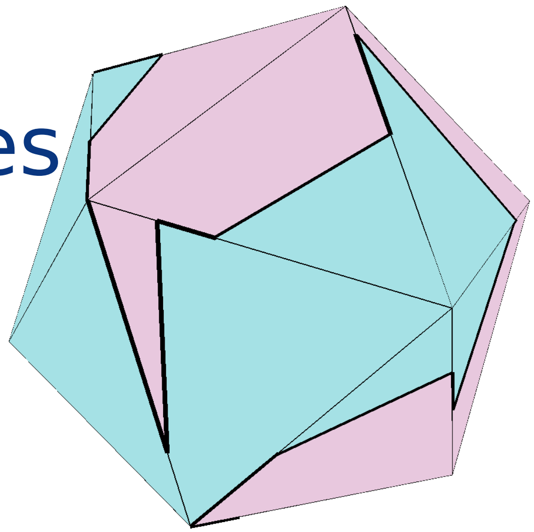
Intersections

Surface-surface intersection is difficult

- complex intersection curves
- singularities, robustness

Instead…

- robust mesh intersection
- symbolic perturbation for topological consistence

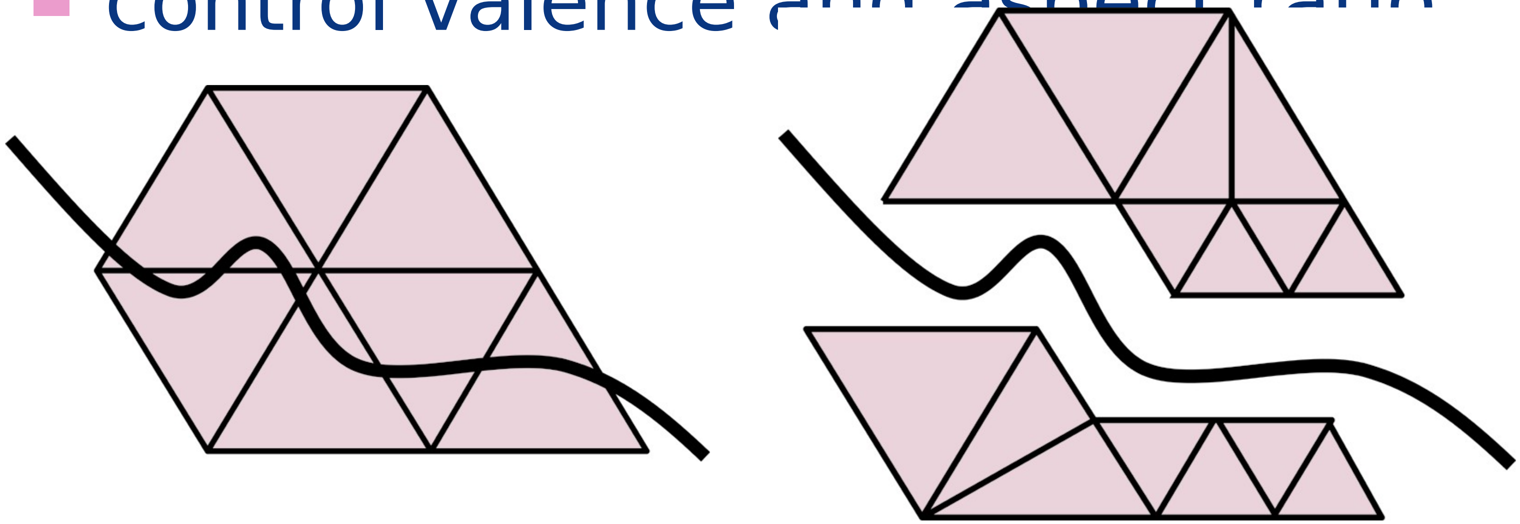


intersection of identical solids

Cut meshes

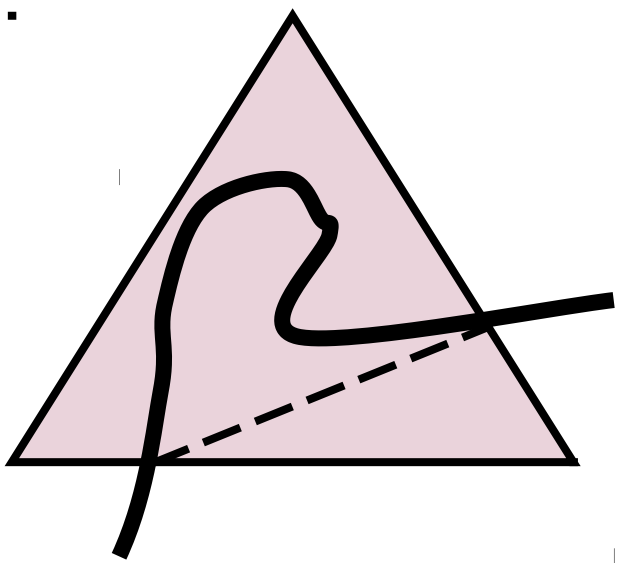
Cut meshes along straight edges

- refine to resolve curve topology
- control valence and aspect ratio

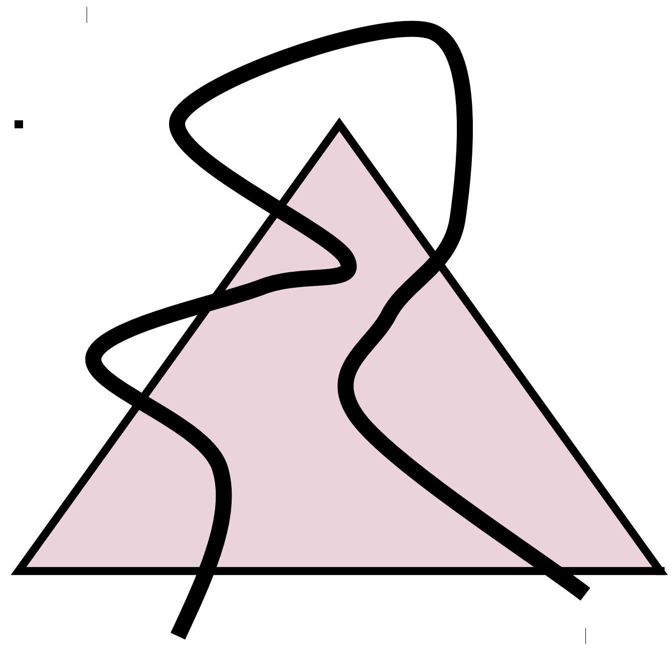


Resolve topology

Avoid refinement if possible



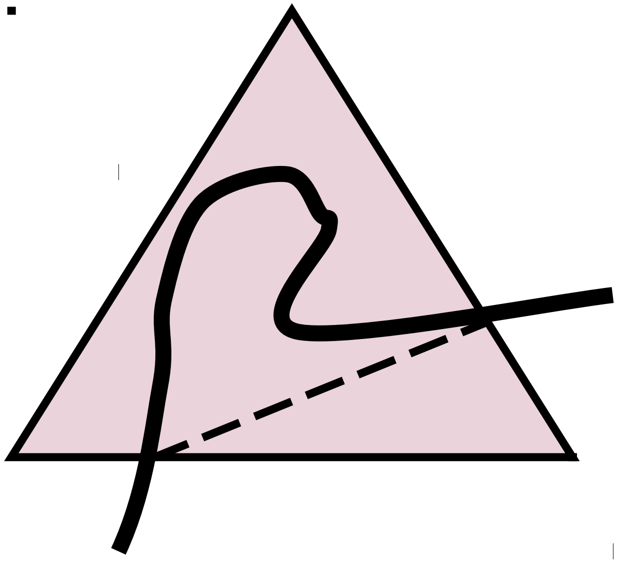
do not refine



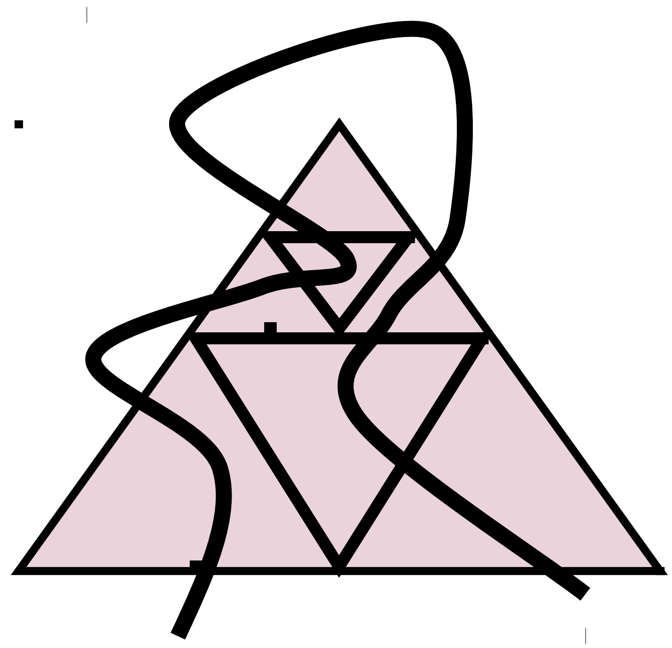
refine

Resolve topology

Avoid refinement if possible



do not refine



refine

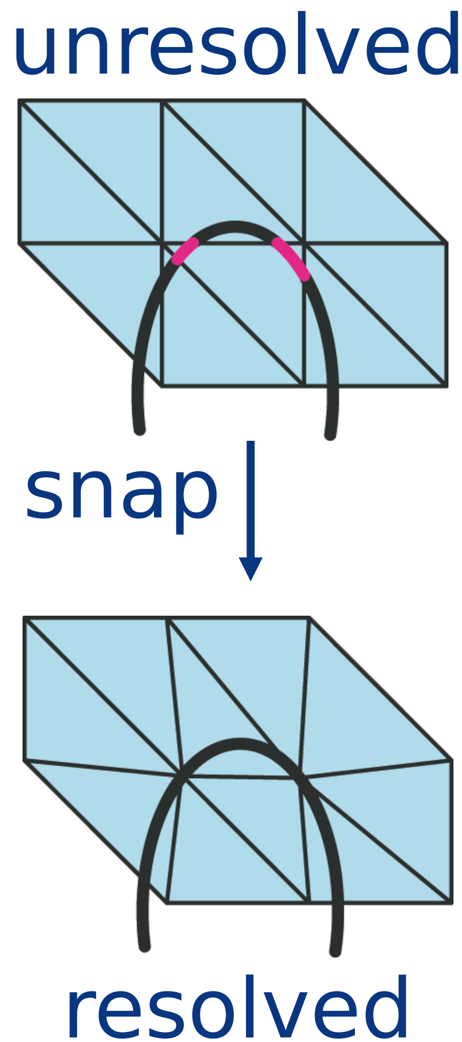
Resolve topology

Snap heuristic

- short curve segments may require many refinement steps
- snap mesh to curve to simplify intersection

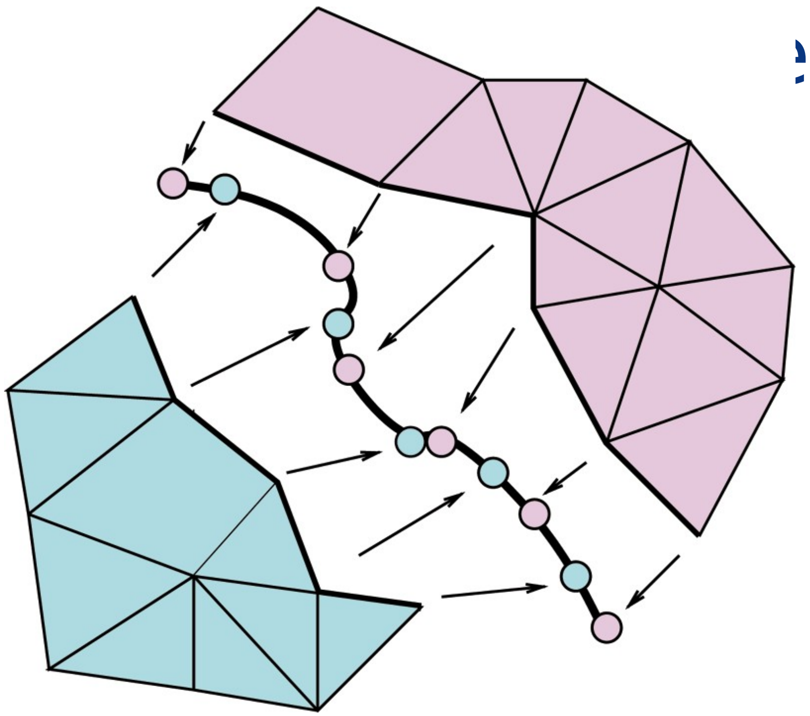
Algorithm

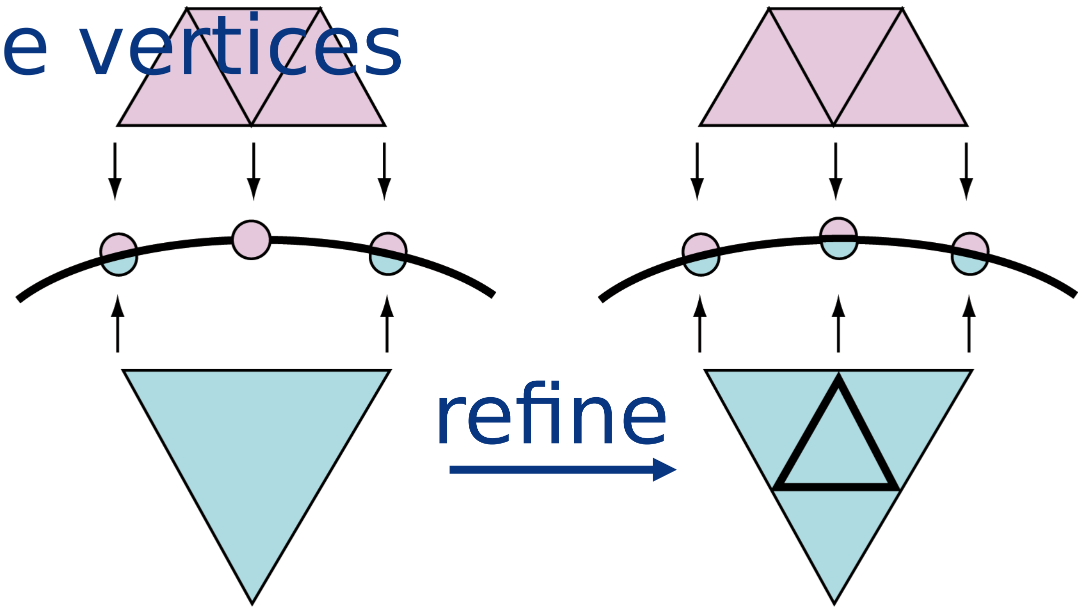
- snap mesh & refine
- repeat until topology resolved



Merge meshes

- Match vertices intersection curve
  - one-dimensional problem
  - eate vertices





Overview again

4 main steps

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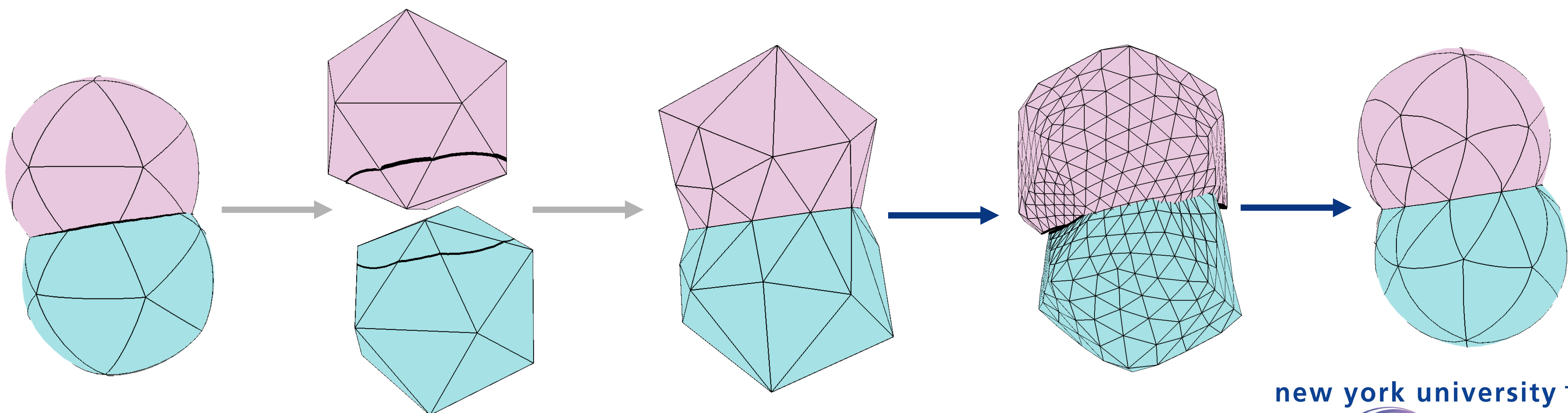
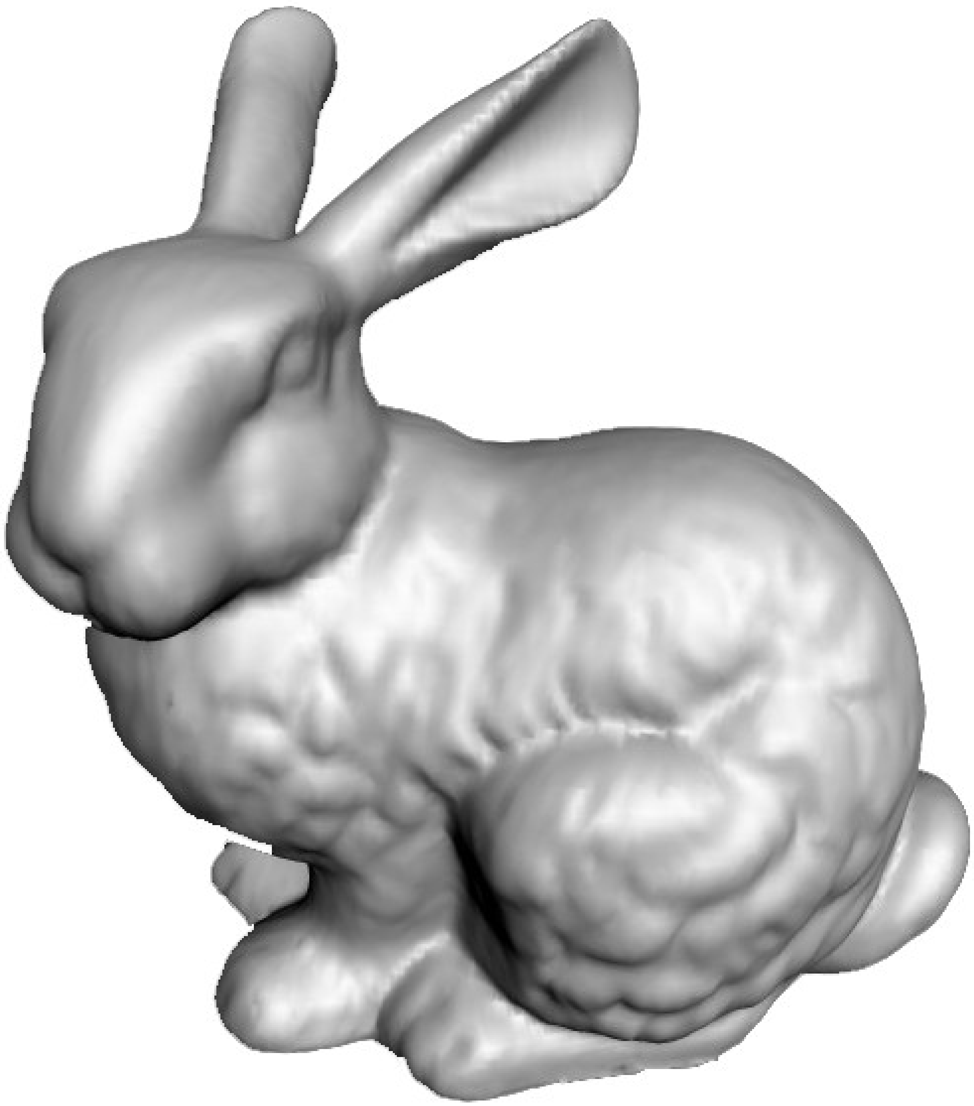
- cut and merge meshes
- parameterization
- fitting

} topology

} geometry



Topological Sphere



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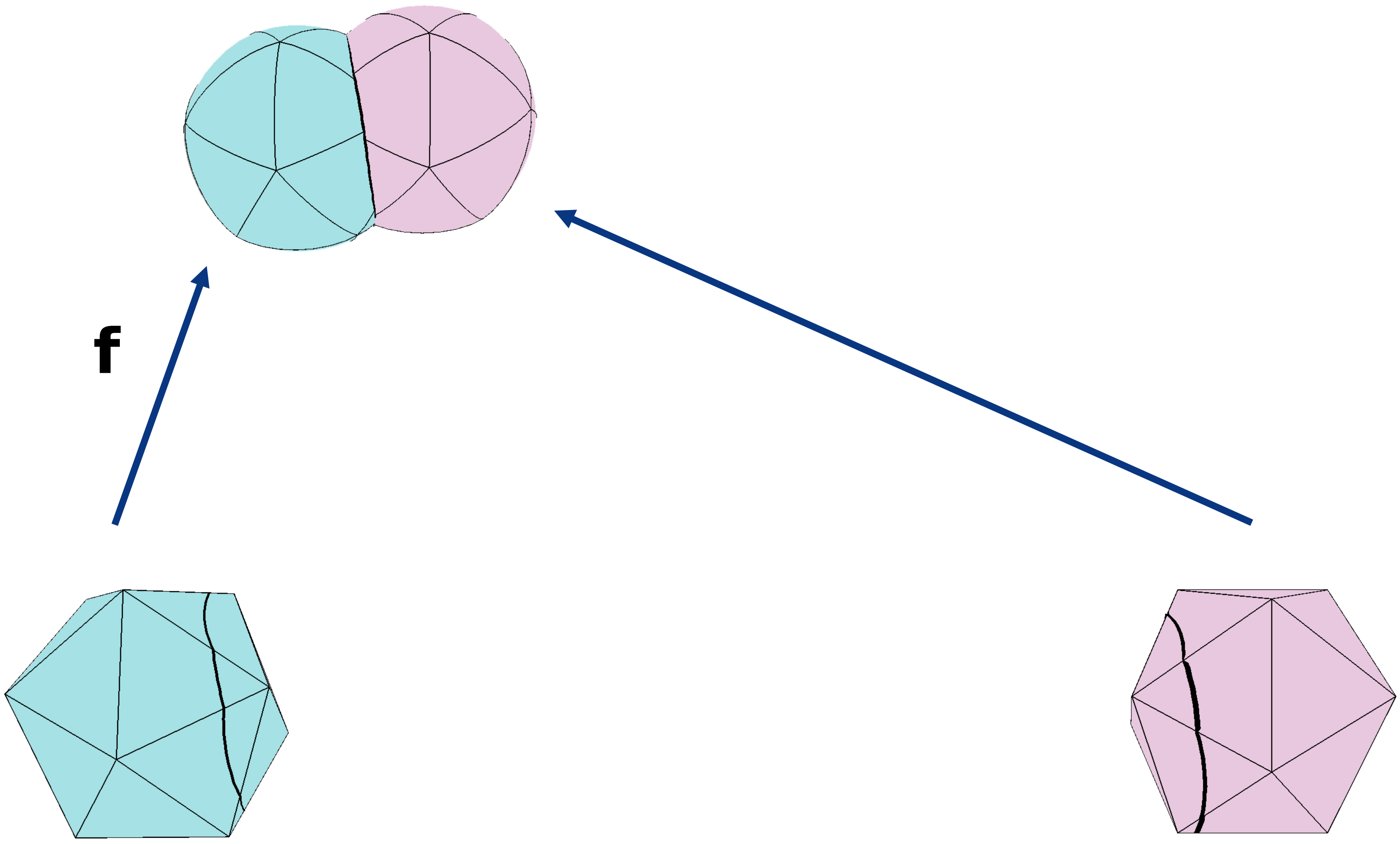
Fitting



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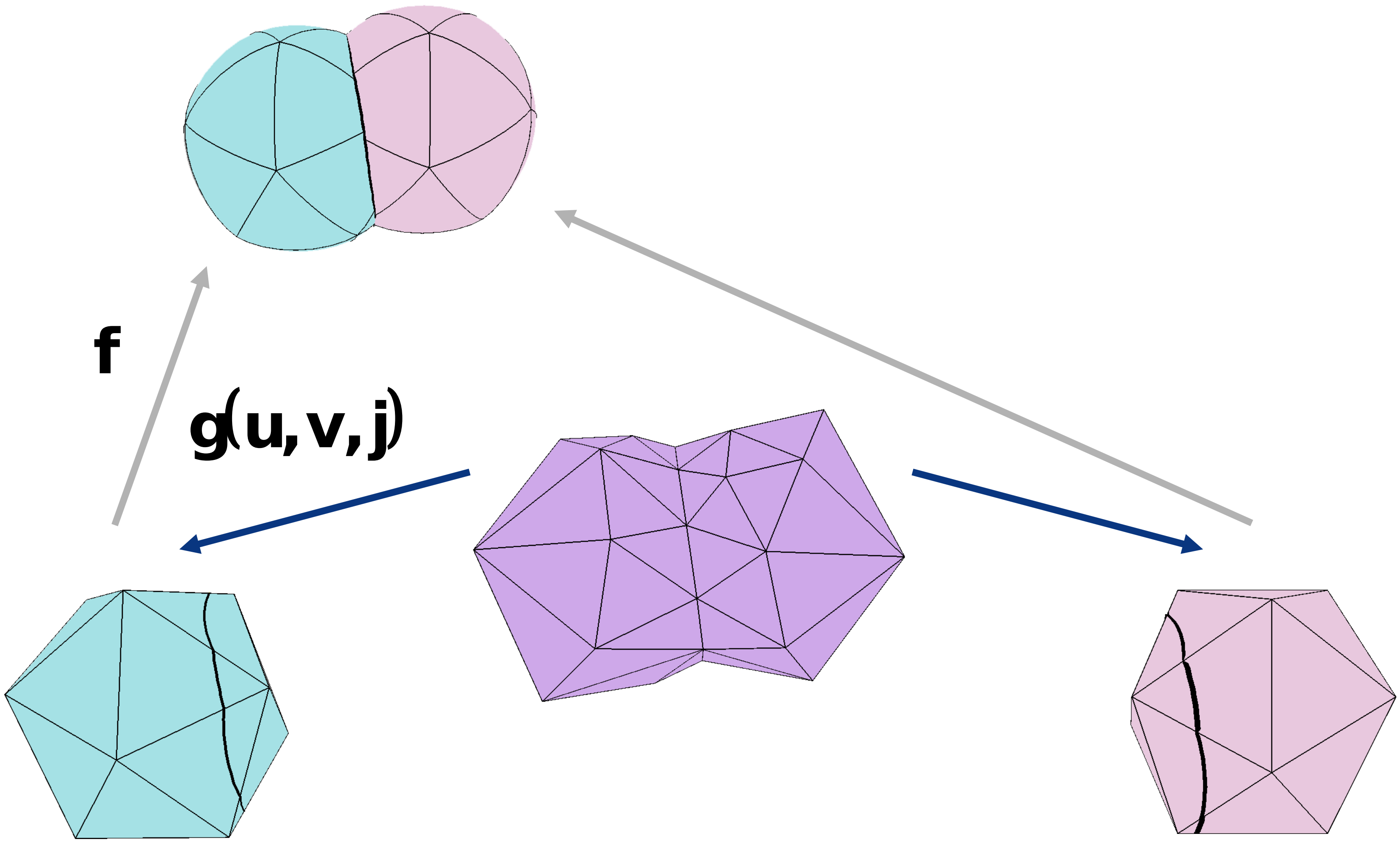
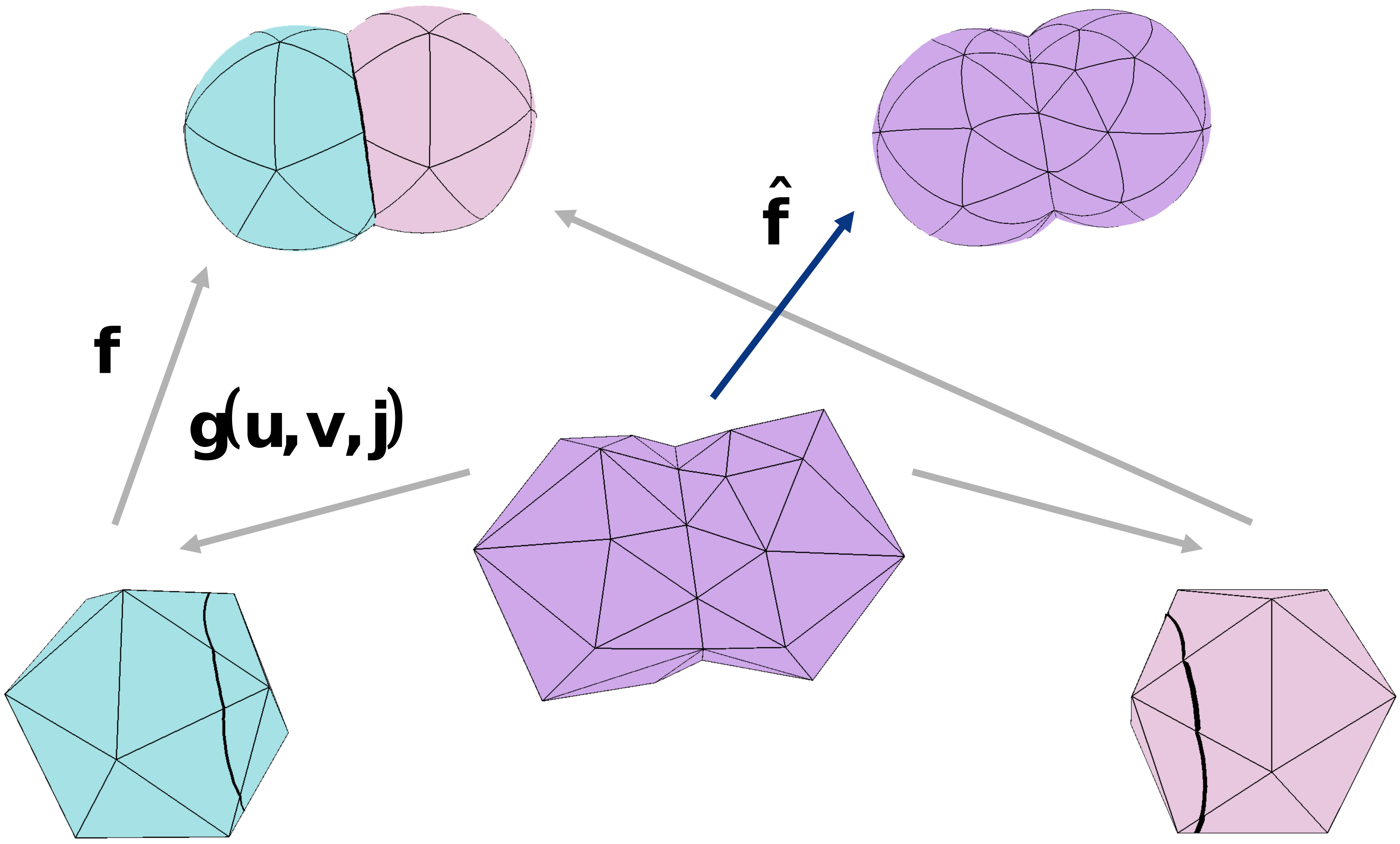
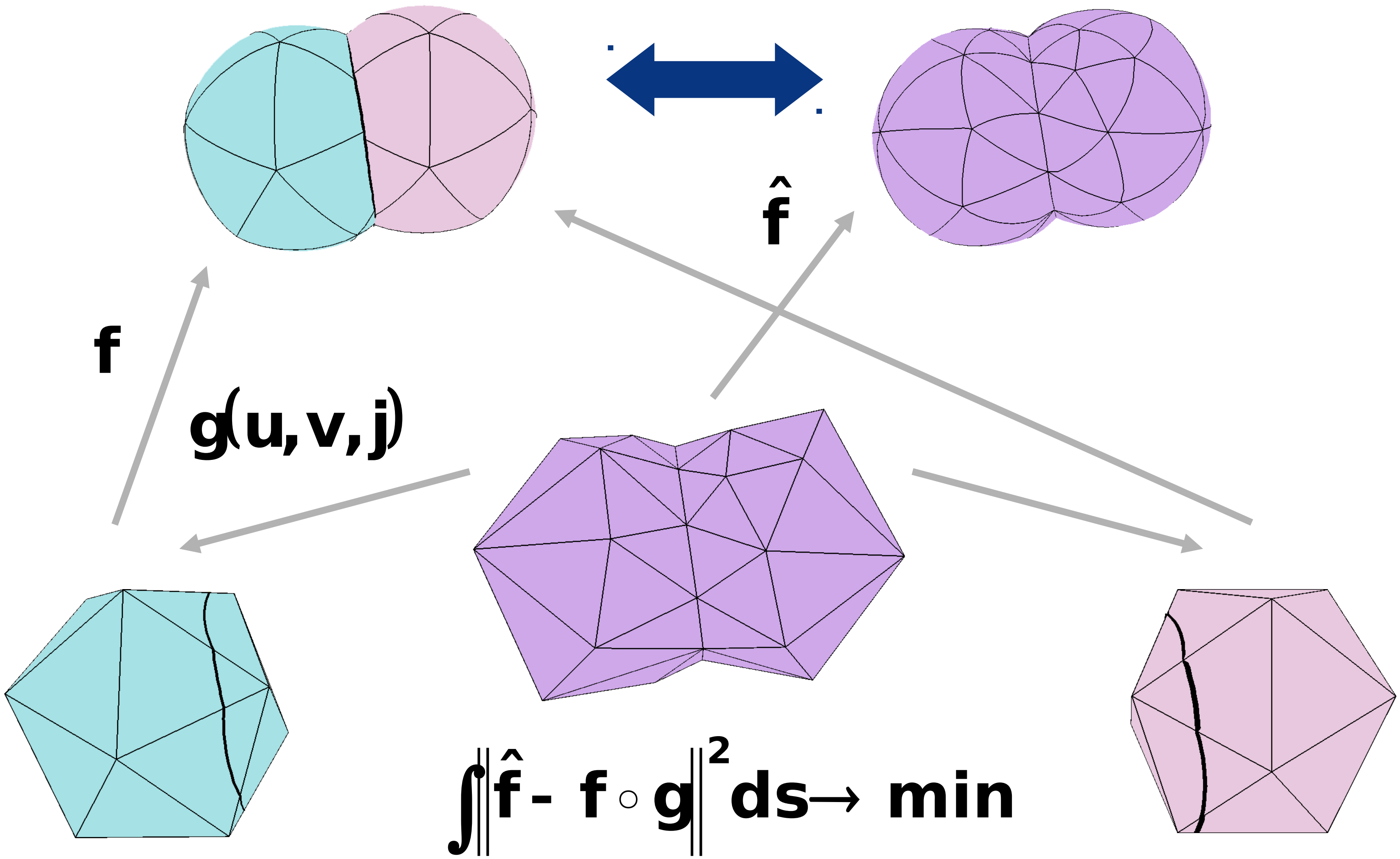


Fitting



Fitting

minimize difference



$$\int \left\| \hat{f} - f \circ g \right\|^2 ds \to \min$$

Fitting

- original surface $\mathbf{f}$
- new surface $\hat{\mathbf{f}}$
- parameterization $\mathbf{g}$
- minimize: $\int \left\| \hat{\mathbf{f}} - \mathbf{f} \circ \mathbf{g} \right\|^2 ds =$

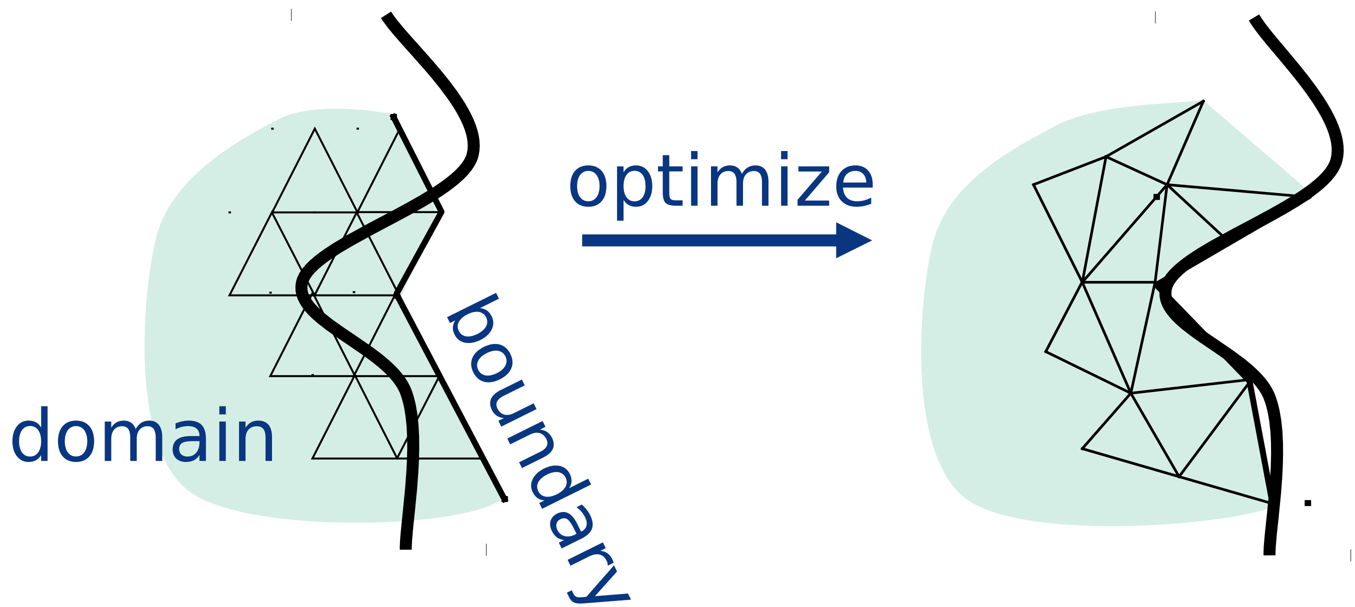
$$\int \left\| \sum_i \mathbf{p}_i \mathbf{B}_i(\mathbf{u}, \mathbf{v}, \mathbf{j}) - \mathbf{f}\left(\mathbf{g}(\mathbf{u}, \mathbf{v}, \mathbf{j})\right) \right\|^2 ds$$

control points

parameterization

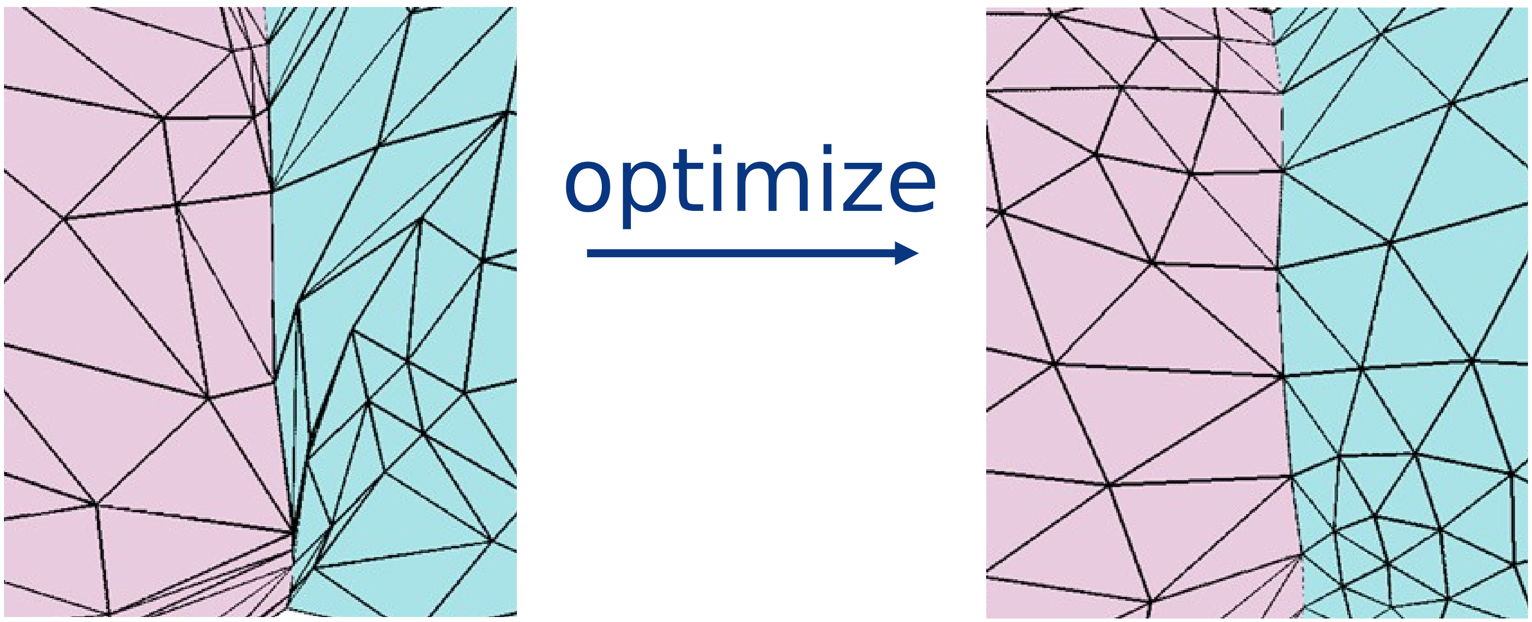
Parameter optimization

- mesh boundary not aligned with intersection curve
- idea: slide mesh along surface



Parameter optimization

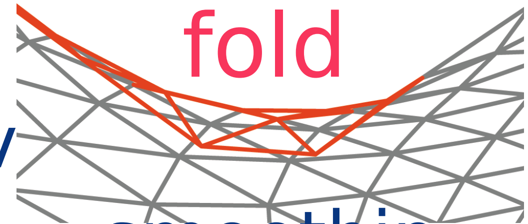
- move vertices in domain
- optimize parameters
- goal: no folds, fair sampling



Parameter optimization

- Laplacian smoothing

  fast, but folds at concav

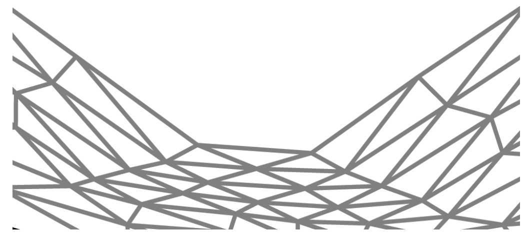


smoothing

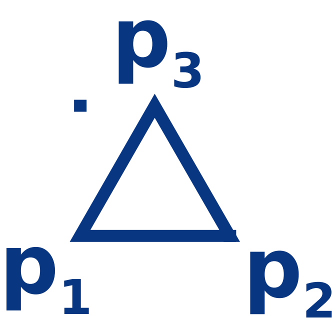
- area/perimeter optimization

  penalize folds

  improve aspect ratios

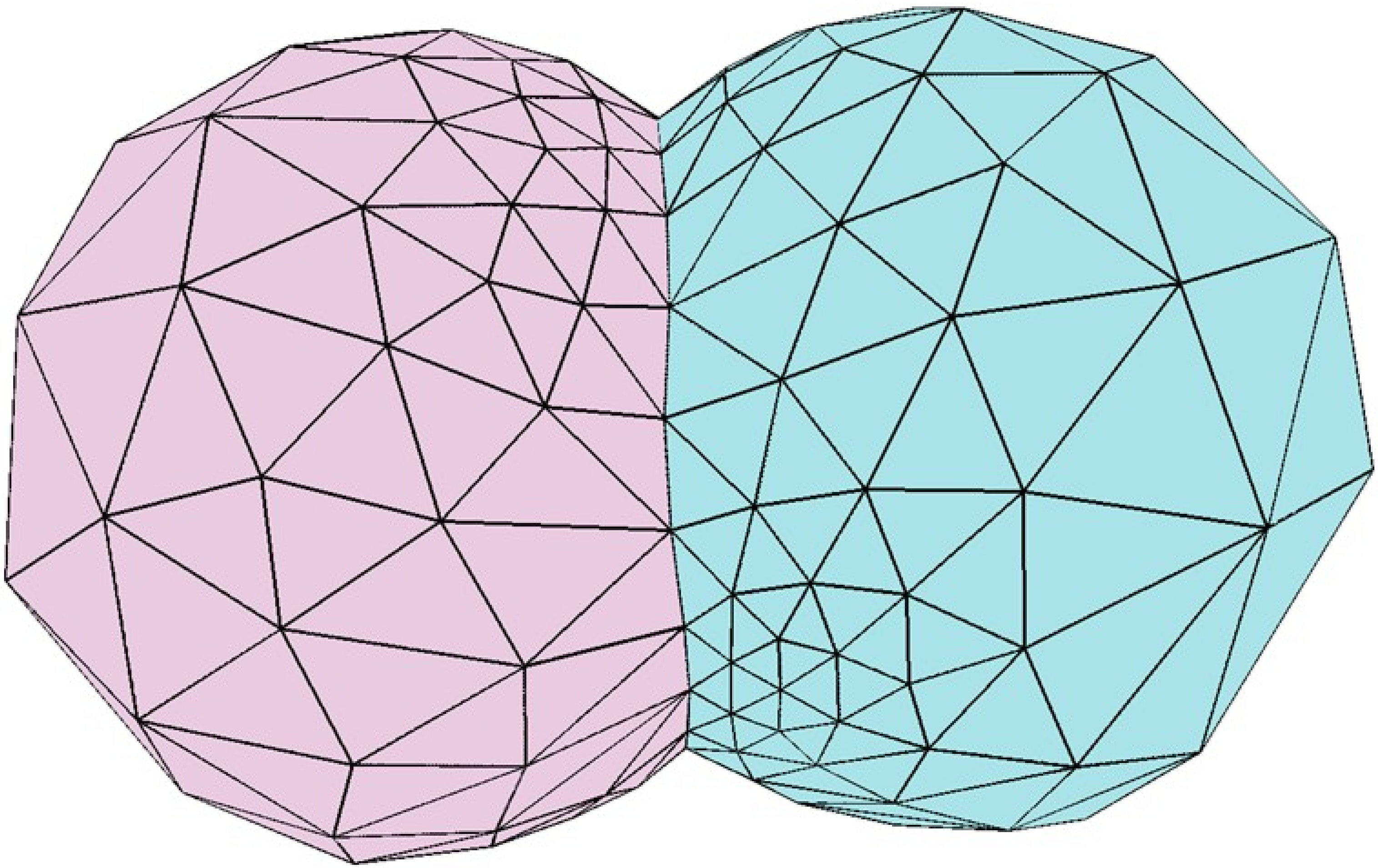


optimization



$$E_{123} = -\frac{\mathrm{Area}(p_1 p_2 p_3)}{p_1^2 + p_2^2 + p_3^2}$$

Smooth domain



Optimize geometry

Control point positions

- least squares fit: sparse system

$$\int \left\| \sum_i \mathbf{p}_i B_i(u,v,j) - f(g(u,v,j)) \right\|^2 ds$$

- better approximation

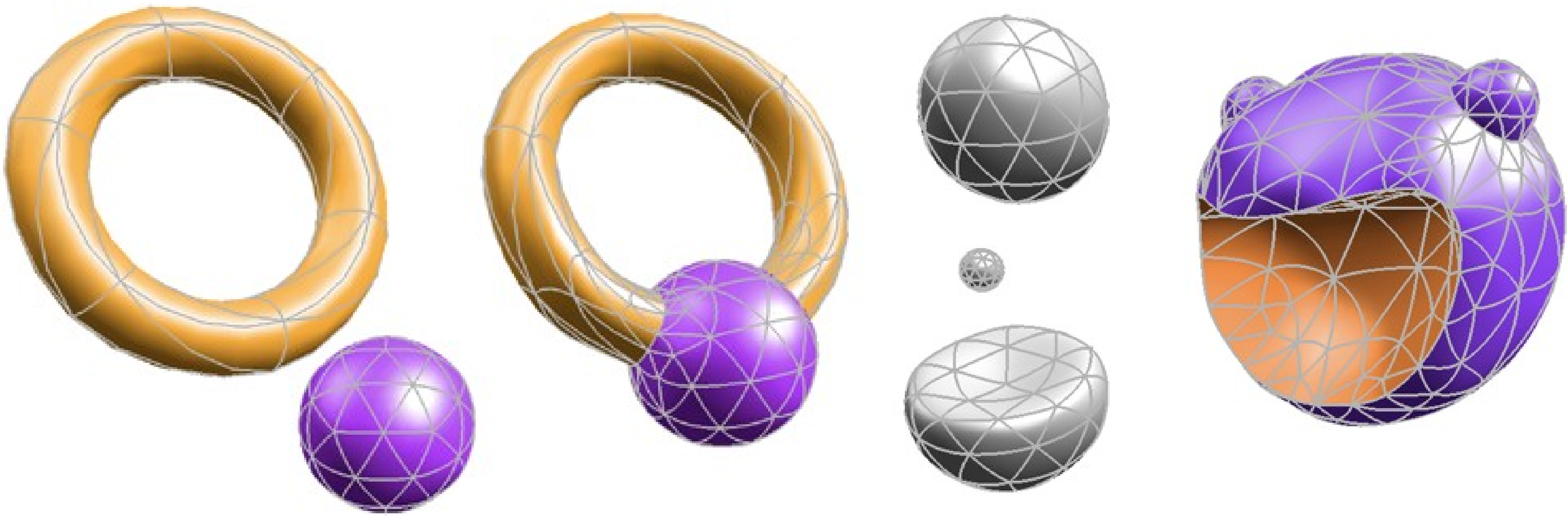
Hierarchical fitting

fitting for $\mathbf{p}^{l+1}$

detail offset: $\mathbf{d}^{l+1} = \mathbf{p}^{l+1} - S\mathbf{p}^{l}$

Results

Small increase in the number of patches in simple cases



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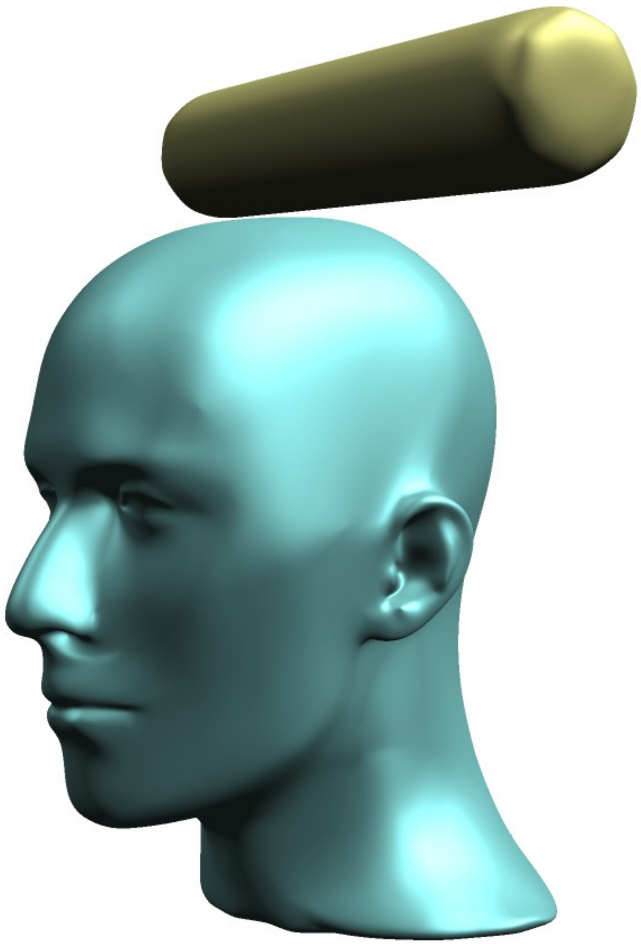
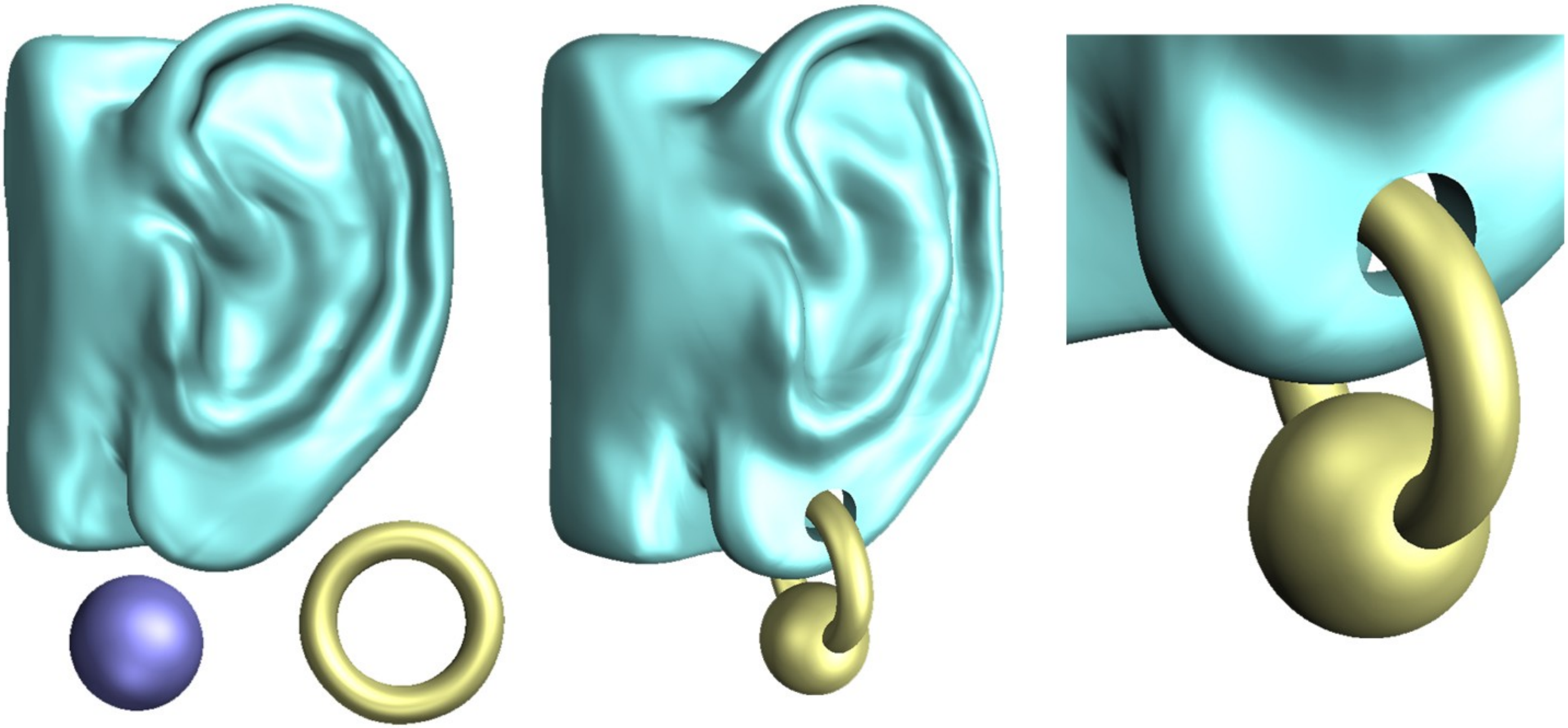


Results

Inspired by S. Dalí’s Venus with Drawers

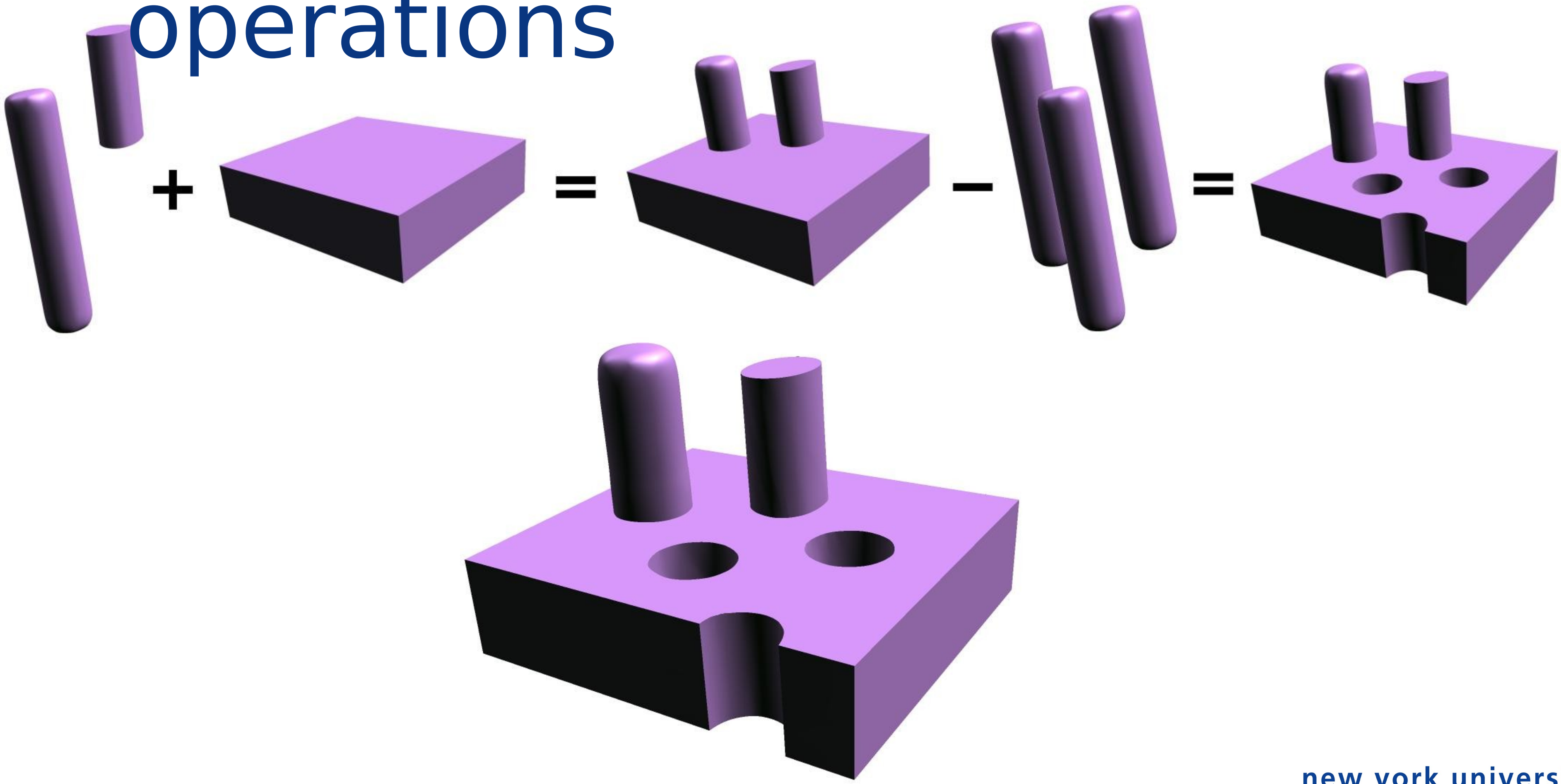
Results

Multires subdivision surfaces



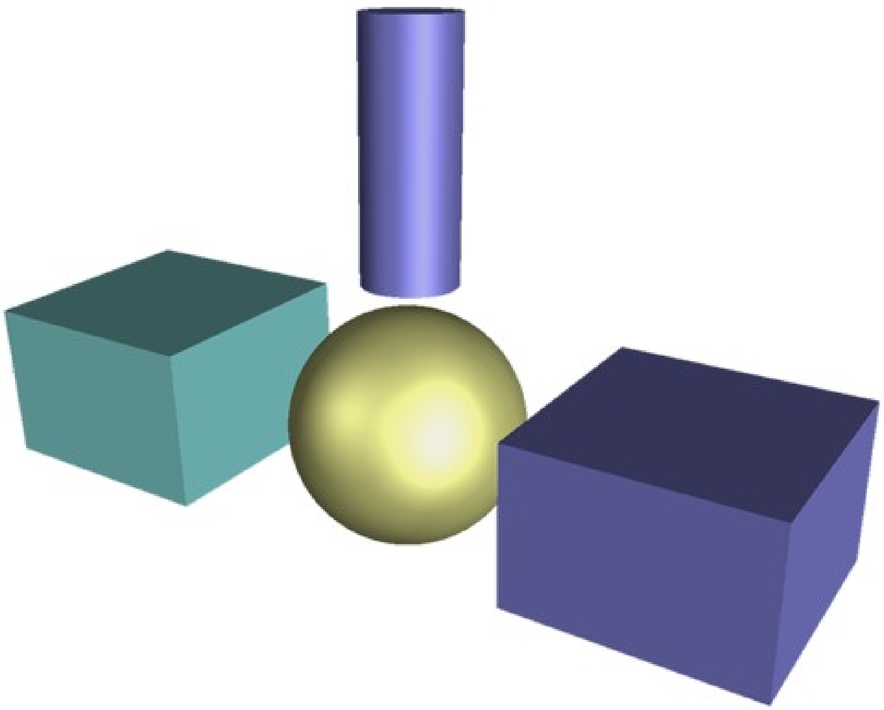
Results

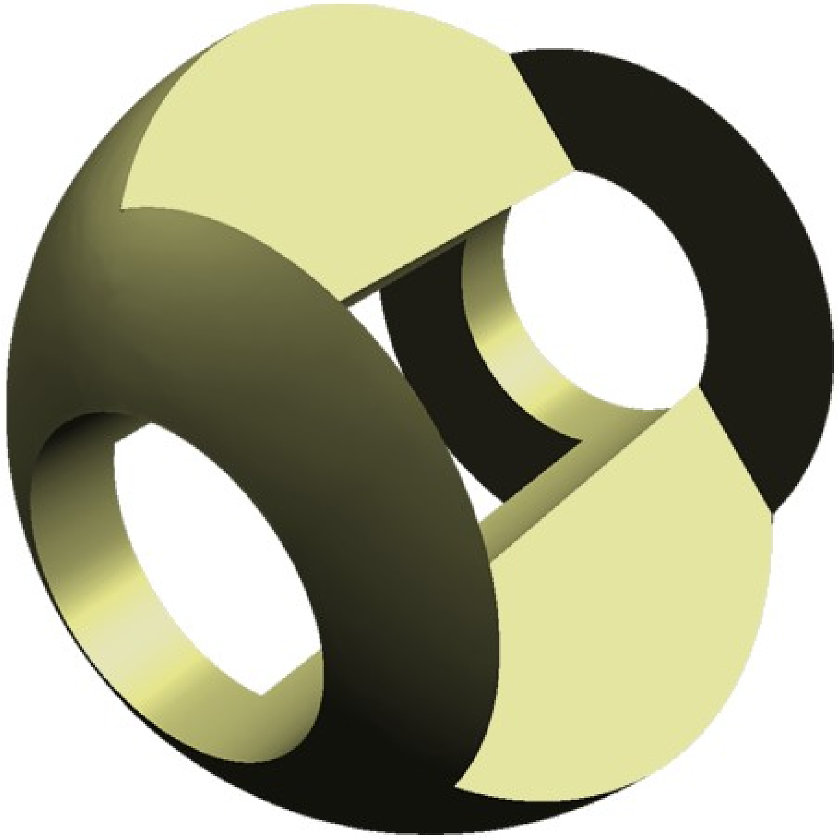
Sequences of Boolean operations



Results

Piecewise-smooth solids





Conclusion

Contributions

- approximate Booleans
- graphics, design applications

Future work

- better parameterizations
- coarsening to get fewer patche
- approximation estimates
- combine with an accurate surface-surface intersection